Projections in Context

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Abstract

This examination considers projections from three space into two space, and in particular their application in the visual arts and in computer graphics, for the creation of image representations of the real world. A consideration of the history of projections both in the visual arts, and in computer graphics gives the background for a discussion of possible extensions to allow for more author control, and a broader range of stylistic support. Consideration is given to supporting user access to these extensions and to the potential utility.

Keywords: Computer Graphics, Perspective, Projective Geometry, NPR

1 Introduction

For tens of thousands of years, human beings have been attempting to represent the world around them in various pictorial forms. Today, denizens of the modern world are veritably bombarded with these depictive images in their day-to-day lives. We have learned how to readily conceptualise, realise, and interpret the transformation of our three dimensional world[1] into a two dimensional representation. This was certainly not always the case – the problem of creating such meaningful representations is complex, and the relative ease with which we can do so today is a tribute to the great minds who have addressed the problem in the past, not a testament of the problem's triviality.

When it came to implementing projections for the purposes of creating images in computer graphics, computer scientists naturally drew on the existing body of work that had been laid down to teach students of art about the principles of projection. This was necessary – as computer scientists learned to put images on the screen, they had to learn the fundamental formulae of the subject, so to speak. There is, however, a much wider variety of experimentation with these principles, and fundamental formulae in the visual arts than has yet been allowed for in computer graphics.

The potential for development here is very great. One of the most inhibiting factors for more experimentation with projective representation is the technical difficulty in the more mathematical aspects of projection. This is exactly where computers can be of great assistance to the authors of representations. It is difficult enough to get all of the angles right in a planar geometric projection. Getting the curves right in a non-planar projection requires a great deal of skill. An algorithm, however, could provide a great deal of assistance. A few computer scientists have realised this, and begun work on developing a broader conceptual framework for the implementation of projections, and projection-aiding tools in computer graphics.

This examination considers this problem of projecting a three dimensional phenomenon onto a two dimensional media, be it a canvas, a tapestry or a computer screen. It begins by examining the nature of the problem (Section 2) and then look at solutions that have been developed both by artists (Section 3) and by computer scientists (Section 4). Finally, discussion of future directions for slightly more grounded conjecture on the subject is included in Section 5, and conclusions based on no results whatsoever are offered in Section 6.

[1]This is not to imply that the universe really is merely three dimensional, only that we perceive it as such. This generalisation will be made throughout this document.

2 The Trouble with Projections

Projecting from three dimensions into two dimensions is a problematic business, and something that we often take for granted. Our eye has evolved such that we gather and interpret information from the world around us in a particular way. Our retina can be described as, not a plane, but a two manifold embedded in three space, as illustrated in Figure 1. So, in a way, the process of vision involves just such a projection, from three space onto this two-manifold.

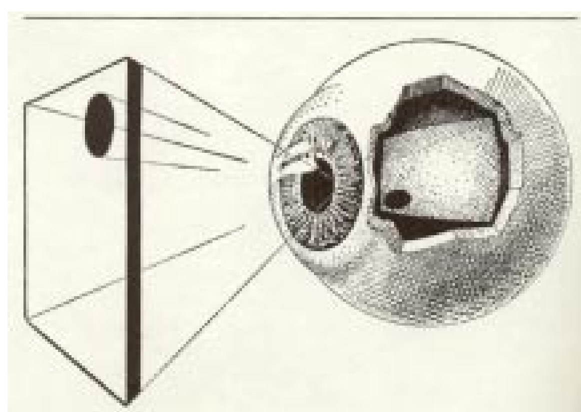

Figure 1: The eye is a spherical screen. Image blatantly plagarised without permission from Flocon and Barre [10].

Still, how can we replicate this process in a manner that allows us to create a representation of the world around us on a plane, or indeed, on any two manifold embedded in three space? Because we have the biological support mechanism for this process (the eye and retina), and the work of many great masters on the subject, it is too easy for us to dismiss this problem as trivial.

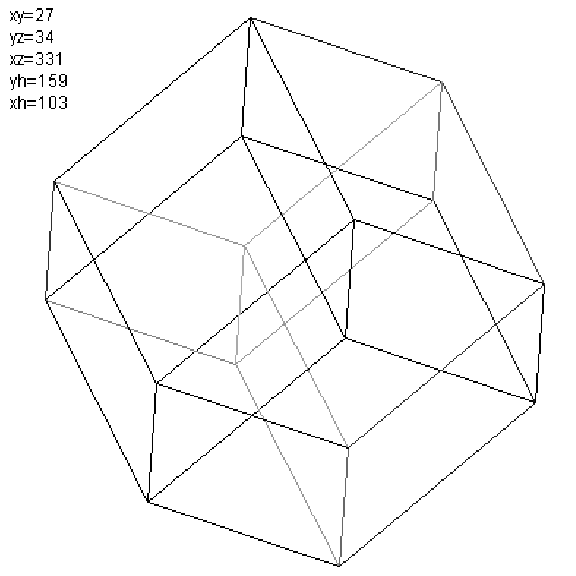

Figure 2: A tesseract projected into three dimensions, and then into two dimensions. It is very difficult to interpret just what is being represented here. Harry J. Smith, http://pw1.netcom.com/ hj-smith/WireFrame4/tesseract.html

In order to dispel this notion of triviality, it may be useful to consider the case of a four dimensional projection into three dimensions. This is not a facility for which evolution has provided humanity with biological support, so we must rely more upon the visualisation. Figure 2 shows a tesseract, or hypercube, projected from four dimensions into three dimensions, and then down into two dimensions. It is extremely difficult to interpret what is going on in this image (at least for this not-so-humble-student).

Providing the ability to rotate the cube, both in the x-z and y-z planes, and the x-h and y-h planes makes things a bit easier – Figure 3 shows the tesseract rotated to nearly 0 in xh, and nearly $\pi/2$ in yh. This is the equivalent, in three space, of looking straight at one face of a cube, such that it looks like a two-dimensional square. Starting from this position, it is easier for us to recognise. Still, the exercise makes a very good point about projections. They are not necessarily simple, or easy or intuitive, and they are certainly not trivial.

Any projection from a higher dimension into a lower dimension (including from three space into two space) is going to be lossy. A point in three space (x, y, z) that is converted into a new point (x', y'), must necessarily lose some information. The problem then becomes the question of exactly what information can be thrown away such that the representation still conveys a desired meaning to its intended audience. The decision about what information can be lost, and what information should be retained really depends on the purpose for which the projection is being made, and the message it is intended to convey.

The aspect of this problem considered in this examination is the question of how computer graphics can give the author of a given representation control over what information can be discarded, what information must be kept, and in what manner the latter is to be used to make the final representation.

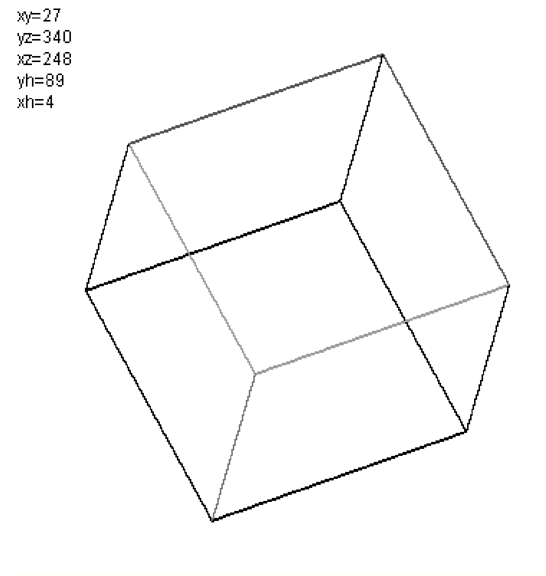

Figure 3: The projected tesseract is rotated such that it resembles a cube in three space. Harry J. Smith, http://pw1.netcom.com/ hj-smith/WireFrame4/tesseract.html

3 Projections in the Visual Arts

One of the greatest problems that people originally faced when trying to figure out just how to project their three dimensional world into two dimensions was understanding – the techniques had not been clearly developed. The other was skill. It is difficult enough to learn to draw projections of three dimensional objects when you have someone to teach you, examples to study, and a great deal of literature on the topic[4].

In this section, we consider the development of projective techniques in the visual arts, paying particular attention to the great variety of techniques that are available. Although there are now many formulaic "how to" manuals[2] available for beginners, the manner in which accomplished artists develop their own projections is hardly so.

3.1 Eastern Tradition

The application of projections in the eastern tradition is a less studied phenomenon than the western tradition, at least less studied in literature available in the languages known to this author, given the time available for this examination. However, this is not to say that eastern artists were less skilled in the use of perspective than their western counterparts, or less creative in their application of projective techniques.

Artist David Hockney makes some consideration of the question of perspective in his studies of eastern scroll paintings [11]. These scrolls can be as nearly a tenth of a kilometer long. This is an extreme sort of medium, and it required an extreme sort of solution to the problem of projecting from three dimensions into two dimensions.

As a scroll unfolds, there are many different viewpoints, and many different projection techniques are thus necessarily employed. The artist would transition from one to another, sometimes

[2]The term *formulaic* is in no way meant as dismissive. As mentioned, learning to draw in perspective is a complex process, and methods for teaching the fundamental *formulae* are necessary, and creatively well developed [7, 2, 25]

Figure 4: Japanese Scroll Fragment. The lower view shows the lines of perspective drawn over the picture. The picture contains multiple viewpoints – one off to the right, and one off to the left. Hishikawa Moronobu, *Scenes in a Theatre Tea-House (fragment)* , The British Museum.

even showing one object from two different view points at once. Figure 4 shows an example of this sort of transition. The right hand side of the depiction (including the building and the right hand wall) is viewed from the right, while the left hand wall is viewed from the left hand side. These two perspectives are blended such that there is no jarring discontinuity, and a tree cleverly covers the in between area where confusion might arise.

3.2 Western Tradition

While there was some sparse use of perspective projection, as we presently understand the term, in classical times, the techniques were either lost or unused for the largest part of the medieval period[4]. This doesn't mean that distant objects were not being represented, it only means that the methods used to project these objects into two dimensions were different. Figure 5 shows an example of the style of projections that were commonly used. The more distant side of the fence in this image is in no way foreshortened. By its higher position on the canvas, its connectedness with the rest of the fence, and its occlusion by the lower fence and the unicorn, the viewer can deduce the fence's depth information [20, 21].

During the Renaissance, the notion of infinitely distant vanishing points re-emerged. Masters like Filarete, Francesca, da Vinci and Durer began experimenting with what we now understand as perspective projection. Each of these masters had his (or in rare but notable cases, her) own techniques, and the variety of techniques was bounded only by the skills and imaginations of artists who were experimenting with them [6, 9, 12, 24].

Figure 6 shows a simple instructive plate done by one of these renaissance artists, Jan Vredeman de Vries. This illustration shows us what is basically our modern concept of three point perspective. While de Vries clearly understood this concept for simple models, he found a much more complicated view was necessary in most practical situations. Figure 7 is perhaps the most famous of all of de Vries' plates – a relatively simple illustration, and already the lines of perspective and foreshortening are getting extremely complex. Nearly every object in this scene has its own vanishing point. The only common factor is that all the vanishing points terminate on the horizon.

Figure 5: Medieval wool warp. Depth information about the back of the fence is conveyed by the position on the canvas (higher vs. lower), the connectedness with the rest of the fence, and with occlusion by the unicorn, and the front part of the fence. Artist Unknown, *Unicorn in Captivity* , wool warp, wool, silk, silver and gilt wefts, 368 cm x 251.5 cm, Metropolitan Museum of Art

3.3 Modern Experiments

The popularisation of photography in the nineteenth century led to a sort of soul searching in the art world. What could artists show, that photographers could not? Photography was cheaper, faster, and more accurate than anything that even the most studied and accomplished traditional artists could hope to replicate. Traditional visual

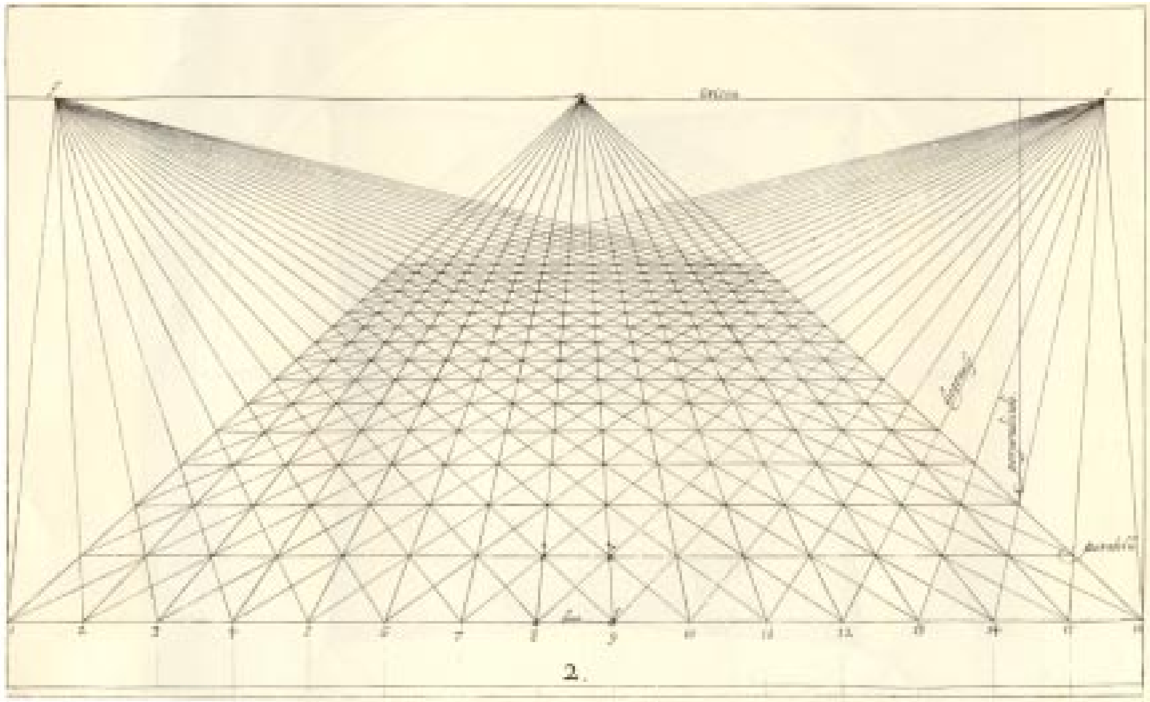

Figure 6: Renaissance Engraving. Depiction of a simple form of perspective projection. Jan Vredeman de Vries, *Perspective No. 2*, Engraving, 1568.

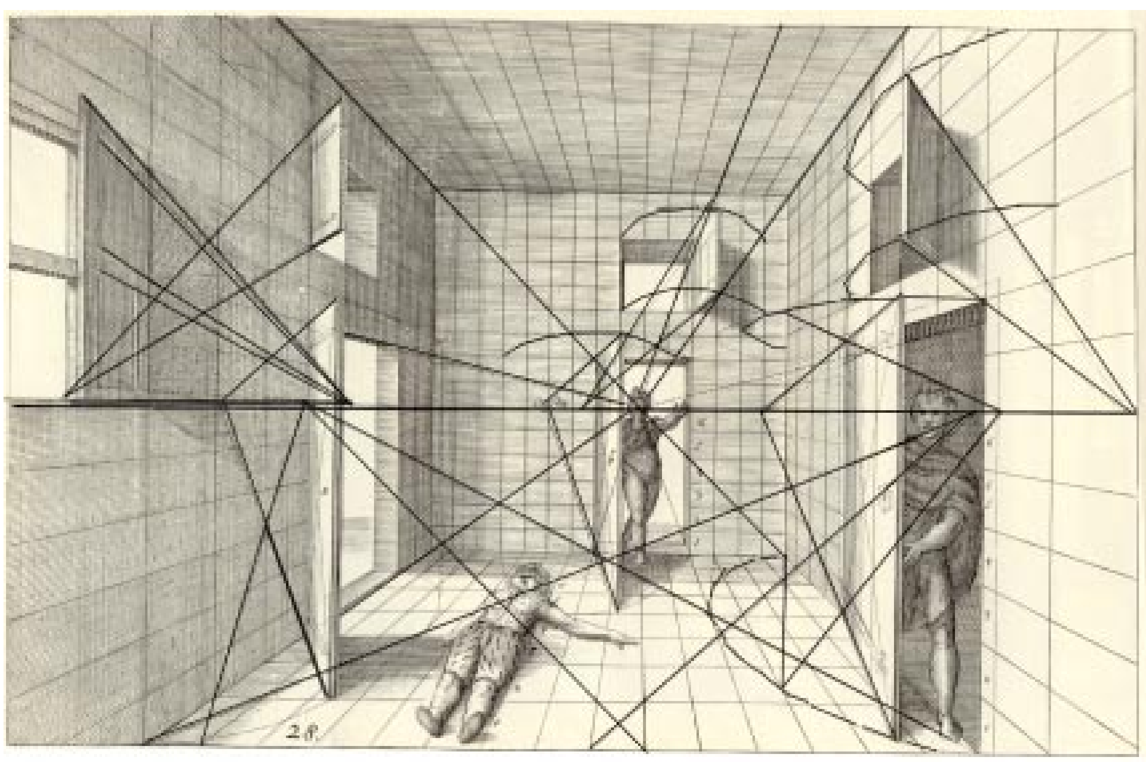

Figure 7: Renaissance Engraving. Artist's horizon lines of perspective have been thickened for presentation here. Even in this relatively simple demonstration, they are quickly becoming extremely complicated. Jan Vredeman de Vries, *Perspective No. 28*, Engraving, 1568.

artists (to say photographers were/are not artists would be both inaccurate, and a disservice) believed they still had a unique way of representing the world, and they wanted to show it [5, 16].

The impressionist movement was one of the primary beneficiaries of this soul searching. It would probably be inaccurate to say that impressionism would not have existed if photography had not been developed. None the less, the popularity of the impressionist movement probably owed much to photography, and the art world's reaction to it. The very world "impressionism", came from an early painting by the artist Claude Monet entitled *Impression: Sunrise*. It shows what Monet, and what later practitioners were getting at – they wanted to convey more in their representation of reality than a mere flat image – they wanted to convey an *impression* of that reality.

The impressionists used light colours and a spontaneous style, much in contrast to the more rigorous *academic* style that had been previously dominant. Many impressionist works are accomplished with a loose collection of brush strokes, in which the viewer must pull the subject out of a seemingly chaotic canvas. As an intentional side effect of this mental composition process, the interpretive functions of the human brain allow an almost three dimensional conception of the subject of an impressionist painting.

This is an approach to projection that more or less tricks the brain into interpreting the three dimensional information. While it is worth mentioning in the context of representing three space in two space, it will not be further considered here, since this discussion will limit itself to geometric projections that can be modelled in computer graphics.

Figure 8: Cubist portrait. The face, suit and hands of the individual are quite recognisable, as well a still life to his right, even though they are all depicted from different perspectives. Pablo Picasso, *Daniel-Henry Kahnweller*, 1910, Oil on canvas, 101.1 cm x 73.3 cm, Art Institute of Chicago.

After impressionism and post-impressionism had past, many artists again began to rethink the question of just how to convey a representation of three dimensions on a two dimensional canvas. Out of this consideration came Pablo Picasso and Georges Braque, and the notion of cubism. Cubism demonstrated that it was possible, and even desirable (depending on one's artistic tastes), to show a completely unrealistic representation of the three dimensional world that still conveyed meaning[3, 17].

Figure 8 shows an example of a cubist portrait by artist Pablo Picasso. The facial features, suit and hands of the individual who is portrayed are all evident, especially when viewed from several meters away, as originally intended. As well, it is possible to make out a still life to the subject's right hand side. Yet, all of these elements are portrayed from entirely different points of view, and placed disjointly onto the canvas where the artist felt it was appropriate.

Figure 9: Cubist painting. The architecture of this structure is not shown in a realistic manner, but distorted to convey the impression desired by the artist. Robert Delaunay. *Saint Severin No. 1*. 1909, oil on canvas, 116 cm x 81 cm, Private Collection, Switzerland.

Not all cubist depictions are disjoint. The principles of the movement are also illustrated in Figure 9, where the architecture of the church of Saint Severin is curved and distorted to impact the viewer in the manner desired by the artist. This is in no way a formulaic projection of three dimensional space onto the two-dimensional canvas. It is a creative development of those traditional techniques to convey a desired impact.

Even with impressionism and cubism, the practice of projection in the visual arts is hardly settled. In the middle of the twentieth century, artist M. C. Escher experimented extensively with warping and twisting the traditional rules of perspective[8, 23]. Figure 10 shows an example of this. While each local part of the image seems to make sense, the whole picture does not fit together. Figure 11, a study done by Escher for the lithograph shown in Figure 10 reveals

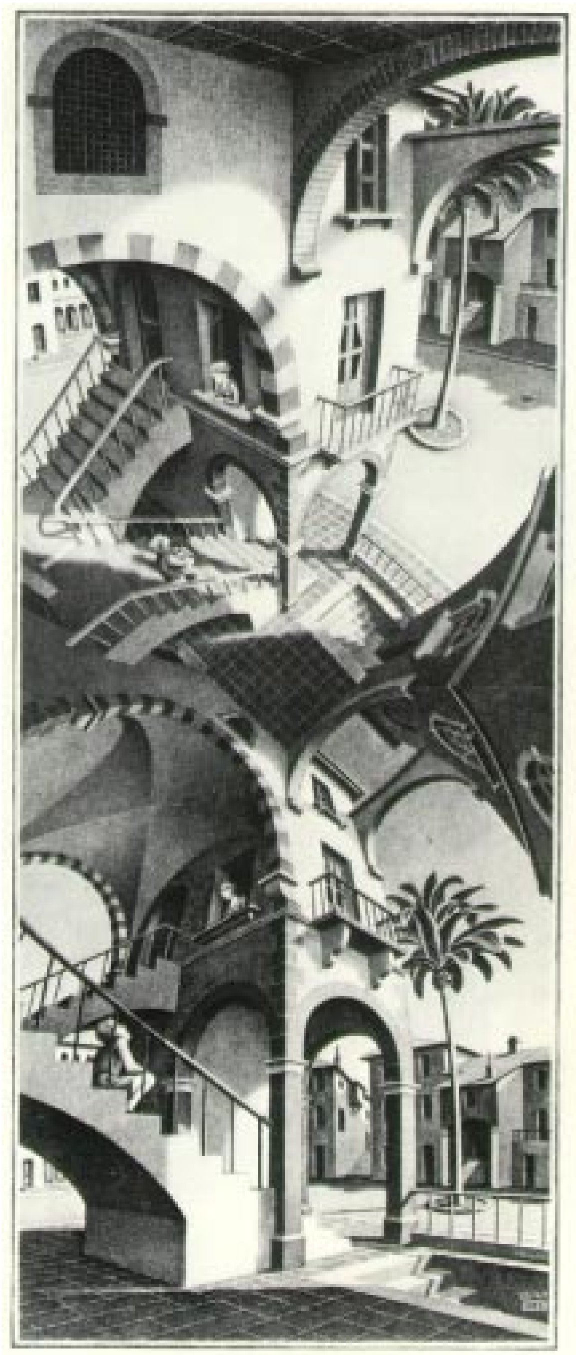

Figure 10: MC Escher, *House of Stairs*, Lithograph, 1951, Lithograph, Catalogue 099, 1968 catalogue.

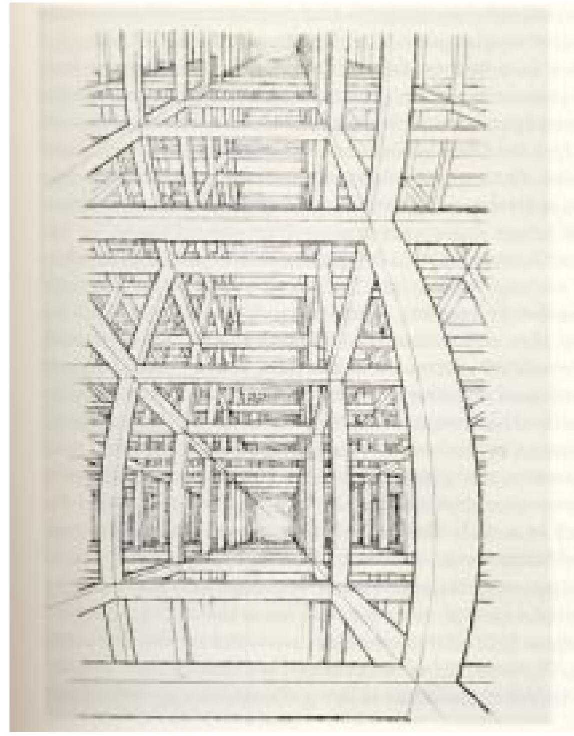

Figure 11: MC Escher, *Study for a Lithograph, Staircase*, 1951, Ink, Catalogue 099b, 1968 catalogue.

the technique that Escher used to create this effect. He has warped the projection in such a way that objects that should be behind the viewer are actually located in front. Figure 12 shows another print by Escher in which a similar technique is used to include multiple viewpoints on a single object in one image. This figure will be discussed more in later sections.

Escher and the cubists are in no way the last artists to play with the projections from three dimensional objects onto a two dimensional canvas. The photo-realists have also done so – taking reality and manipulating it to their liking. Figure 13 shows an example of a photo-realist painting by Richard Estes. The distortion applied by Estes is most visible in the bus at the left centre of the painting [14, 15].

In summary, consideration of how to represent three dimensional space in two dimensions is still underway in the visual arts. Experimentation with different projective techniques has been going on since classical times, and shows no sign of stopping in the future.

4 Projections in Computer Graphics

The application of perspective in computer graphics has long been considered a solved problem. Indeed, the mathematical underpinnings of currently used projections were in place long before the first electronic computers were even developed.

4.1 Standard Applications

The mathematical theory of perspective was investigated by Gerard Desargues as early as 1639. Further developments were made in the next century by Gaspard Monge, who is considered to be the "father of descriptive geometry". By 1864, which marked the publication of Albert Church's often-cited *Elements of Descriptive Geometry* , the discipline of descriptive geometry was well founded [4].

Figure 13: Photo-realist painting. The projective distortion that is applied by the artist is most clearly visible in the bus at the centre right of the painting. Richard Estes, *Park Row Looking Toward City Hall* , 1992, oil on canvas, Louis K. Meisel Gallery.

One of the earliest pieces on the application of projective geometry to computer graphics was a Ph.D. thesis by Lawrence G. Roberts, actually in Electrical Engineering. In his thesis, Roberts develops algorithms for a simple viewing transformation which involves dividing a point's coordinates by its distance from the viewpoint. Roberts applies homogeneous coordinates to this problem, and gives a matrix representation of the transformation[19].

Many developments and more than a decade later, Ingrid Carlbom and Joseph Paciorek published their seminal paper *Planar Geometric Projections and Viewing Transformations*. In this paper, they concisely ordered and documented many of the commonly used planar geometric projections, and illustrated how these projections could be accomplished with homogeneous transformation matrices. Their ordering of projections is shown in Figure 14.

A planar geometric projection is defined by a projection plane and a set of projectors from that plane out into the world. Points in the space being projected are drawn at the point at which the projector which intersects them meets the projection plane.

All of the planar geometric projections that are described here can be accomplished in computer graphics by means of a translation, a rotation and a projective transformation. Thus, given a point Q, the projected point Q' can be found by $Q' = P \cdot R \cdot T \cdot Q$, where P is the projection matrix, R is the rotation matrix, and T is the translation matrix. Since R and T are the same for any projection type, and are well studied in most introductory graphics text books, we consider them only briefly. To translate the projection plane to the origin, it is only necessary to find the plane's normal, and the closest point on the plane to the origin. The translation can then be done as follows, where (n_x, n_y, n_z) represents the projection plane's normal, and d the distance of that closest point.

$$T = \begin{bmatrix} 1 & 0 & 0 & dn_x \\ 0 & 1 & 0 & dn_y \\ 0 & 0 & 1 & dn_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotation R can be computed in a number of ways. Here, we use spherical coordinates, which seems by far the most elegant method available. Figure 15 shows us a depiction of spherical coordinates. In this system, our point at (x, y, z) is considered as a vector. The symbol θ is used to represent the angle between the x axis and the vector being represented. This can be calculated as $tan^{-1}(y/x)$. Similarly, ϕ represents the angle between the vector and the z axis, and can be calculated as $cos^{-1}(z/\rho)$ where ρ is the length of the vector (denoted as $\hat{r}$ in Figure 15).

$$\begin{aligned} x &= \rho cos(\theta) sin(\phi) \\ y &= \rho sin(\theta) sin(\phi) \\ z &= \rho cos(\theta) \end{aligned}$$

The rotation R then becomes a rotation about the z axis by $-\theta$, and a rotation about the y axis by $-\phi$. We can compose this by multiplying the two standard matrices for such transformations together, as follows:

$$\begin{aligned} R &= \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 & 0 \\ sin(\theta) & -cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\phi) & 0 & sin(\phi) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\phi) & 0 & cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} cos(\theta)cos(\phi) & -sin(\theta) & cos(\theta)sin(\phi) & 0 \\ sin(\theta)cos(\phi) & cos(\theta) & sin(\theta)sin(\phi) & 0 \\ -sin(\phi) & 0 & cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

We now continue with our discussion of projection. In the discussion below, it may be assumed that the above transformations have already been applied, and we must now consider how to formulate P, our projection matrix.

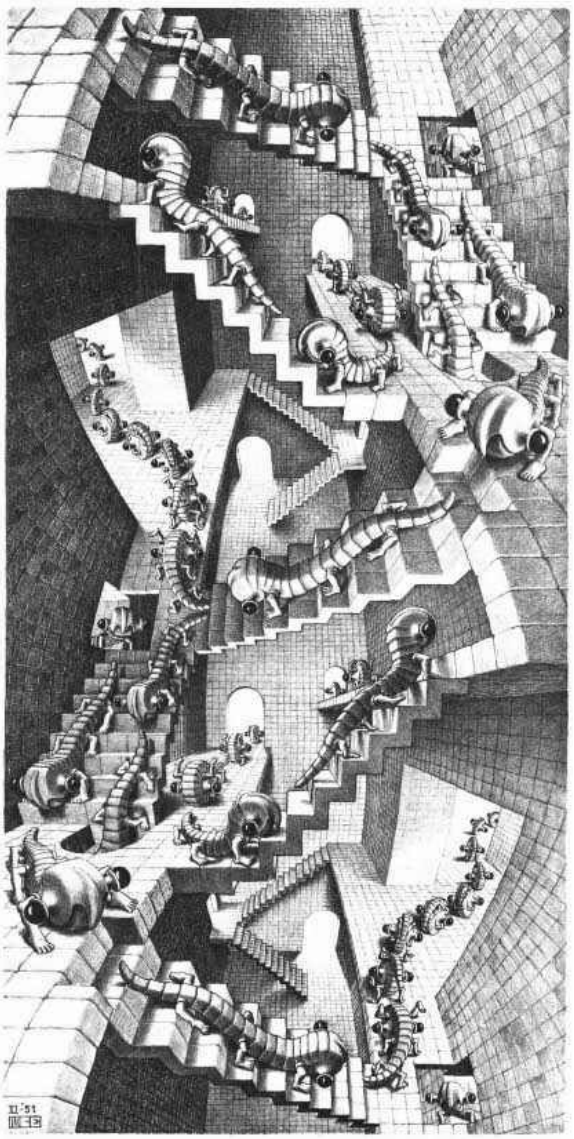

Figure 12: MC Escher, *Up and Down*, 1947, Lithograph, Catalogue 087, 1968 catalogue.

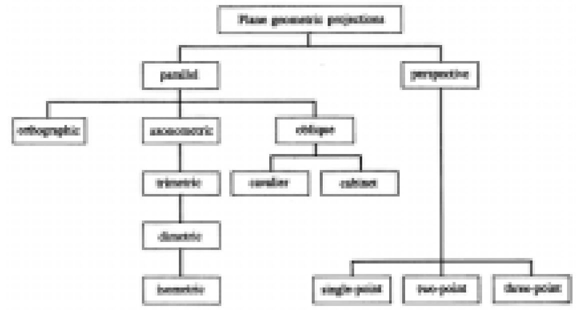

Figure 14: Tree showing the breakdown of planar geometric projections. After Carlbom and Paciorek [4]. Blatantly pilfered from some guy's power point presentation that the author of this examination got off the web and now can't find the URL for to reference properly.

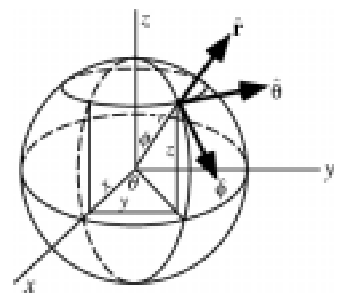

Figure 15: Spherical Coordinates Representation of a Point. Unrepentantly pilfered from MathWorld at http://mathworld.wolfram.com/SphericalCoordinates.html.

4.1.1 Parallel Projections

In a parallel projection, the viewer is located at infinity. As a result, all of the lines of projection are parallel to one another. This is extremely useful in architecture and design, and in other situations where measurements need to be taken from drawings. Since the lines are parallel, it is possible for precise measurements to be made.

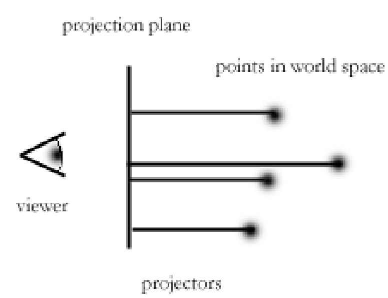

Figure 16: Orthographic projection. The projectors are all parallel to one another and orthogonal to the projection plane.

Orthographic projections are particularly useful for this sort of measurement taking, and they are often given with multiple views (front, top, side, and possibly interior) so that the object(s) being depicted can be entirely reconstructed. Figure 16 shows a graphical representation of this mechanism – notice that the lines of projection are not only parallel to one another, but also perpendicular to the viewing plane.

An orthographic projection can be accomplished quite easily in computer graphics. Once we have accomplished our R and T transformations, as described above, it is a simple matter of eliminating the z coordinate as follows. This is the identity matrix, with the unit value in the row corresponding to z missing.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Axonometric projections are similar in nature to orthographic projections, in that the projectors are parallel to one another, and perpendicular to the axis. Unlike orthographic projections, however, the viewing angle is chosen such that an object being represented is viewed from an angle off of its main axes. Unfortunately, there are problems with this view. Because the human brain is trained to understand perspective projections, it can easily misinterpret this sort of projection. Figure 17 shows one way this can happen. Parallel lines receding into the distance are most readily interpreted to mean the object in question gets larger. A similar problem occurs with circles – they are interpreted as elliptical.

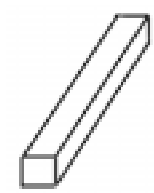

Figure 17: Trimetric axonometric projection

In computer graphics, axonometric projections can be accomplished in the same way as orthographic projections, but with an additional rotation multiplied into the mix to account for the difference in viewing angle.

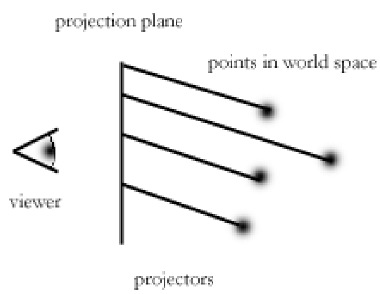

Figure 18: An oblique projection. The lines of projection are parallel, but they are not perpendicular to the projection plane.

Axonometric projections are commonly divided into three categories, based on the angles between the projection plane and the object's main axes. If all three of these angles are equal, projection is called isometric. If two of them are equal, the projection is dimetric. Finally, if none of them are equal, the projection is referred to as trimetric.

Oblique projections also have parallel projectors, but the projectors are not perpendicular to the projection plane. This mechanism is modelled in Figure 18. This projection is very similar to an axonometric projection, and has much the same benefits and issues involved. If the angle is well selected, and the object is carefully positioned according to a known rule-set, it can produce good results, and because the foreshortening ratio (the object's projected length divided by its true length) is uniform, accurate measurements can still be made from drawings in this style.

By convention, oblique projections where the angle between the projection plane and the projectors is $\pi/4$ are called cavalier. The foreshortening ratio of a cavalier projection is one. The object's projected length is the same as it's true length. Oblique projections at an angle of $arccot(1/2)$ (about 63 degrees) are referred to as cabinet projections. Their foreshortening ratio is one half, that is to say that lines not parallel to the view plane are shortened by a factor of one half.

Oblique projections can be accomplished in computer graphics by means of a shearing effect, representing the angle of obliqueness, and then an orthographic projection. This can be represented as follows, where a and b are the shearing factors.

$$P = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.1.2 Perspective Projection

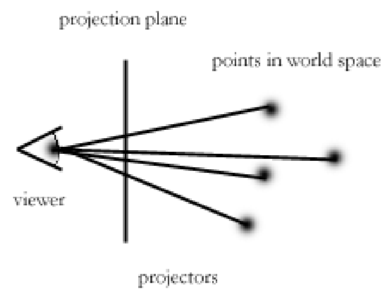

Figure 19: Perspective projection. The projectors are taken as lines from a centre of projection or eye point, to the point being projected.

Of the traditionally used projections, perspective projection is arguably the most realistic. A graphical depiction of the mechanism behind this technique is shown in Figure 19. Because the lines of projection are not parallel in a perspective projection, the foreshortening ratio does not exist, as such. We will be able to see this more clearly once we have worked out the mathematics behind the perspective projection.

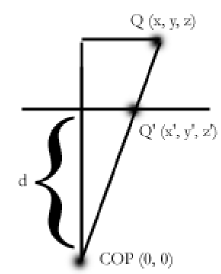

Figure 20: Perspective

In order to calculate the matrix representation of a perspective projection for computer graphics, similar triangles are used, as shown in Figure 20. If we place the centre of projection (COP) at the origin (a simple translation from the position assumed above), we can see that x is to z as x' is to d. To calculate x', then, we cross multiply, and achieve $x' = x * z/d$. We calculate similarly for y', and can derive the following perspective projection matrix, where d represents the distance from the centre of projection to the projection plane.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

It can be seen from this that the foreshortening ratio of a perspective projection is not a constant, but a function. This student has never seen a name given to this function, and so will arrogantly presume to call it a *foreshortening function* , so that it can be referred to later. In this standard perspective projection, the *foreshortening functions* in x, y and z are given by Equation 1.

$$\begin{aligned} f_x &= \frac{z}{d} \\ f_y &= \frac{z}{d} \\ f_z &= 0 \end{aligned} \quad (1)$$

4.2 Recent Developments

Recently, more work has been done on allowing some amount of variation in these traditional techniques. Singh has done work on non-linear perspective in [22], and Agrawala et al have considered the blending of multiple projections in [1]. Wood et al created multi-perspective panoramas, which very much resemble some of the artwork from the eastern tradition. They did this by creating various windows on an animation from different perspectives. The windows were used to create guides, which could be turned into higher quality images by an artist, and then composited together to form a final sequence [26].

Paul Rademacher has developed techniques which allow the actual geometry of an object to change depending on the position of the viewer[18]. These techniques were extended by D. Martin et al, into a system which allowed a variety of warps to be created based on the observer's position in the scene [13].

There is clearly interest in expanding the ability of the artist to manipulate projections. The remainder of this examination will attempt to address the question of where we can go from here – of what can be developed in this area, how it can be developed, and how we can test the utility of these ideas.

5 Future Work

This paper has attempted to present a background to the way in which perspective is currently considered, both in the art world, and in computer graphics. New tools have always given artists new ways of doing perspective, from the camera obscura on. The next question to ask is – how can computers develop new ways of doing perspective, that cannot be done in other media?

6 Conclusions

Every visual medium has its strengths and weaknesses. Computer graphics, as a medium is particularly strong in providing mathematical support to artists for tasks that were previously tedious, or impossible due to the technical challenges involved. Experimentation with projections from three space to two space is just such an area, and the development of tools to support the authors of two dimensional representations of three space could prove extremely valuable.

Although there are pedagogical formulae for these projections in the visual arts, there is also a great deal of experimentation. One of the biggest things holding back this experimentation is in fact the technical difficulty involved in developing new twists on old methods. It took some very smart people, masters like Francesca, da Vinci and de Vries, to come up with the fundamental rules that we use today, though even they had a broader understanding of the topic than we allow for today in computer graphics.

This is certainly an area where computers can help artists out a great deal, providing mathematical support for the foundations of warped and twisted projections.

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